Auctions

CMPS 17

Theresa Migler-VonDollen

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There are 4 main types of auctions in which a single item is sold:

- *Ascending-bid auctions* also called *English auctions*. The seller gradually raises the price and bidders drop out one by one until only one bidder remains. This bidder pays the current price.
- *Descending-bid auctions* also called *Dutch auctions*. The seller gradually lowers the price until a bidder accepts and pays the current price.
- In *first-price sealed-bid auctions* all bidders submit their bid in a sealed envelope. The highest bidder wins the object and pays the price she bid.
- In *second-price sealed-bid auctions* all bidders submit their bid in a sealed envelope. The highest bidder wins the object and pays the price of the second highest bid.

Equivalent types of auctions

Descending-bid auctions and first-price sealed-bid auctions are equivalent.

Ascending-bid auctions and second-price sealed-bid auctions are very similar.

Auctions as Games

- Each bidder is a player.
- The strategies are the bids.
- The payoffs depend on:
  - How you truly value the item (the *true value*).
  - If you win or lose.
  - Which type of auction you are playing in.

Second-price sealed-bid auctions

In this type of auction bidding your true value is the dominant strategy.

Definition

Suppose an item is being auctioned. Let v_i be bidder i's true value for the item and b_i be the bid (strategy) for bidder i.

- If b_i is not the winning bid, then the payoff to bidder i is 0.
- If b_i is the winning bid, and some b_j is the second place bid, then the payoff to bidder i is $v_i - b_j$.

Simple two player second-price sealed-bid auctions

- Suppose that Bidder 1 values the item at $18.
- Suppose that Bidder 2 values the item at $12.
- Bidder 1 bids either $10 or $20.
- Bidder 2 bids either $7 or $17.
- The following payoff matrix illustrates this:

		Bidder 2: $7	Bidder 2: $17
Bidder 1	$10	11, 0	0, 2
Bidder 1	$20	11, 0	1, 0

Second-price sealed-bid auctions

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Definition

Suppose an item is being auctioned. Let v_i be bidder i's true value for the item and b_i be the bid (strategy) for bidder i.

- If b_i is not the winning bid, then the payoff to bidder i is 0.
- If b_i is the winning bid, and some b_j is the second place bid, then the payoff to bidder i is $v_i - b_j$.

Second-price sealed-bid auctions

Theorem

In a second-price sealed-bid auction, it is the dominant strategy for each bidder i to bid $b_i = v_i$.

Proof.

If bidder i does not bid v_i there are two cases:

1. $b_i > v_i$:
   - If b_i doesn't win, then v_i would not have won either. In this case the payoff for both is 0.
   - If b_i does win, there are two cases:
     - v_i also wins, in which case the payoff for v_i and b_i are the same.
     - v_i does not win. The second highest bid must be between v_i and b_i, so the payoff to v_i is 0, but the payoff to b_i is $v_i - b_j \leq 0$.

□

Second-price sealed-bid auctions

Theorem

In a second-price sealed-bid auction, it is the dominant strategy for each bidder i to bid $b_i = v_i$.

Proof.

If bidder i does not bid v_i there are two cases:

1. $b_i > v_i$: Done
2. $b_i < v_i$
   - If b_i doesn't win, there are two cases:
     - v_i also doesn't win, in which case the payoff for v_i is the same.
     - v_i does win. So the payoff to v_i is $v_i - b_j > 0$.
   - If b_i does win, then v_i also wins, both have a payoff of $v_i - b_j$.

In any case it is no worse to bid v_i. □

The intuition behind this is that your bid only determines if you win or lose, not how much you pay.

First-price sealed-bid auctions

In this type of auction bidding your true value is no longer the dominant strategy.

Definition

Suppose an item is being auctioned. Let v_i be bidder i's true value for the item and b_i be the bid (strategy) for bidder i.

- If b_i is not the winning bid, then the payoff to bidder i is 0.
- If b_i is the winning bid, then the payoff to bidder i is $v_i - b_i$.

If bidder i bids v_i and wins, her payoff is still 0.
So bidders "shade" their bids.
Determining how much to shade is complex.

All-pay auctions

Each bidder places a bid.
The winner gets the item AND *everyone* pays their bid.

Definition

Suppose an item is being auctioned. Let v_i be bidder i's true value for the item and b_i be the bid (strategy) for bidder i.

- If b_i is not the winning bid, then the payoff to bidder i is $-b_i$.
- If b_i is the winning bid, then the payoff to bidder i is $v_i - b_i$.

This seems unrealistic, but in fact occurs often in the real world.

- Political lobbying.
- Web design firms trying to win a contract.

The Seller's Perspective

- Suppose that there are two bidders in a second-price sealed-bid auction.
- Bidder 1 bids either 3 or 5, each with probability 1/2.
- Bidder 2 bids either 3 or 4, each with probability 1/2.
- What is the seller's expected revenue?

		Bidder 2	
		$3	$4
Bidder 1	$3	$3	$3
	$5	$3	$4

- Since each value occurs with probability 1/4, the seller's expected revenue is 13/4.
- Note that this is NOT a payoff matrix, it's an expected revenue matrix.

The Seller's Perspective

- Suppose that there are three bidders in a second-price sealed-bid auction.
  - Bidder 1 bids either 2 or 5, each with probability 1/2.
  - Bidder 2 bids either 2 or 5, each with probability 1/2.
  - Bidder 3 bids either 2 or 5, each with probability 1/2.
- What is the seller's expected revenue?

There are 8 possibilities, each occurring with probability 1/8:

2 2 2	2 5 5
2 2 5	5 2 5
2 5 2	5 5 2
5 2 2	5 5 5

- The seller's expected revenue is 28/8.