Graph Theory Basics

CMPS 17

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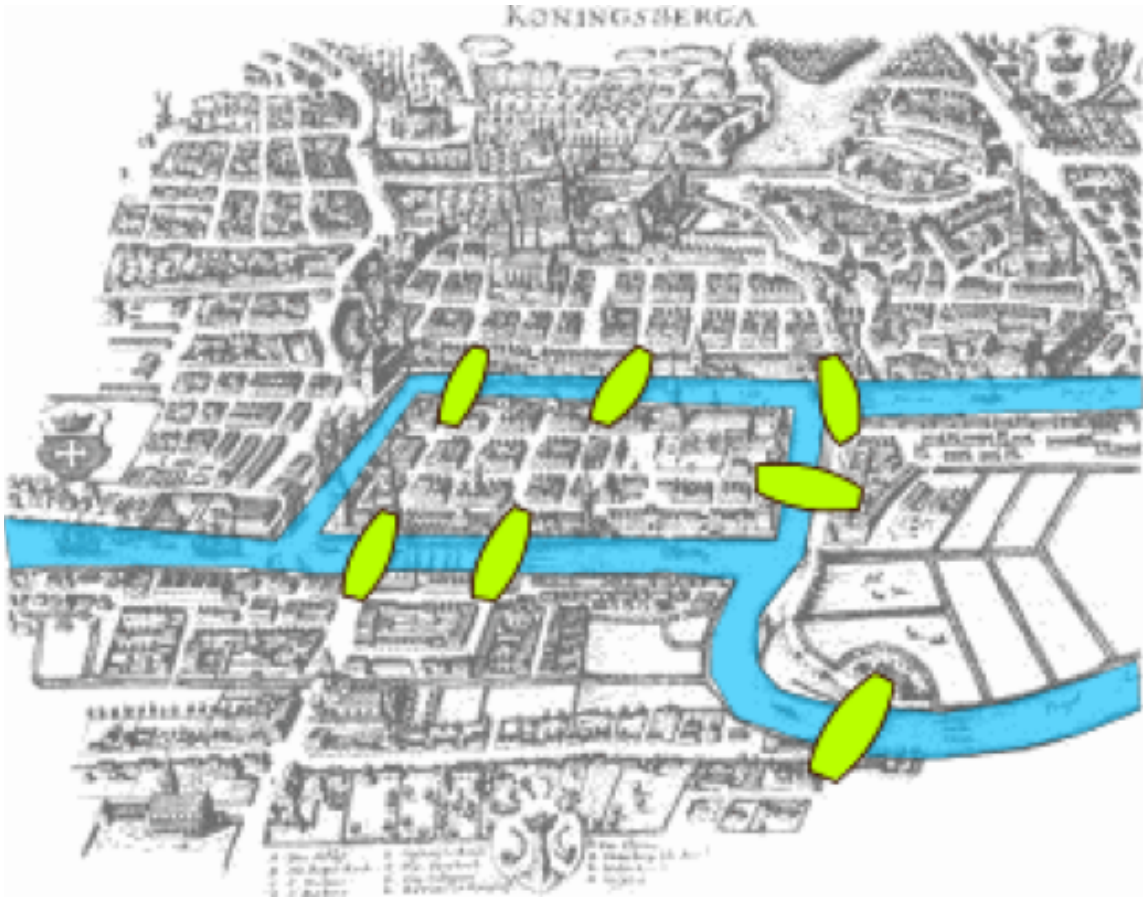
Graph Theory

- Father of graph theory: Leonhard Euler

- Swiss mathematician
- *Seven Bridges of Königsberg* 1736.

Seven Bridges of Königsberg

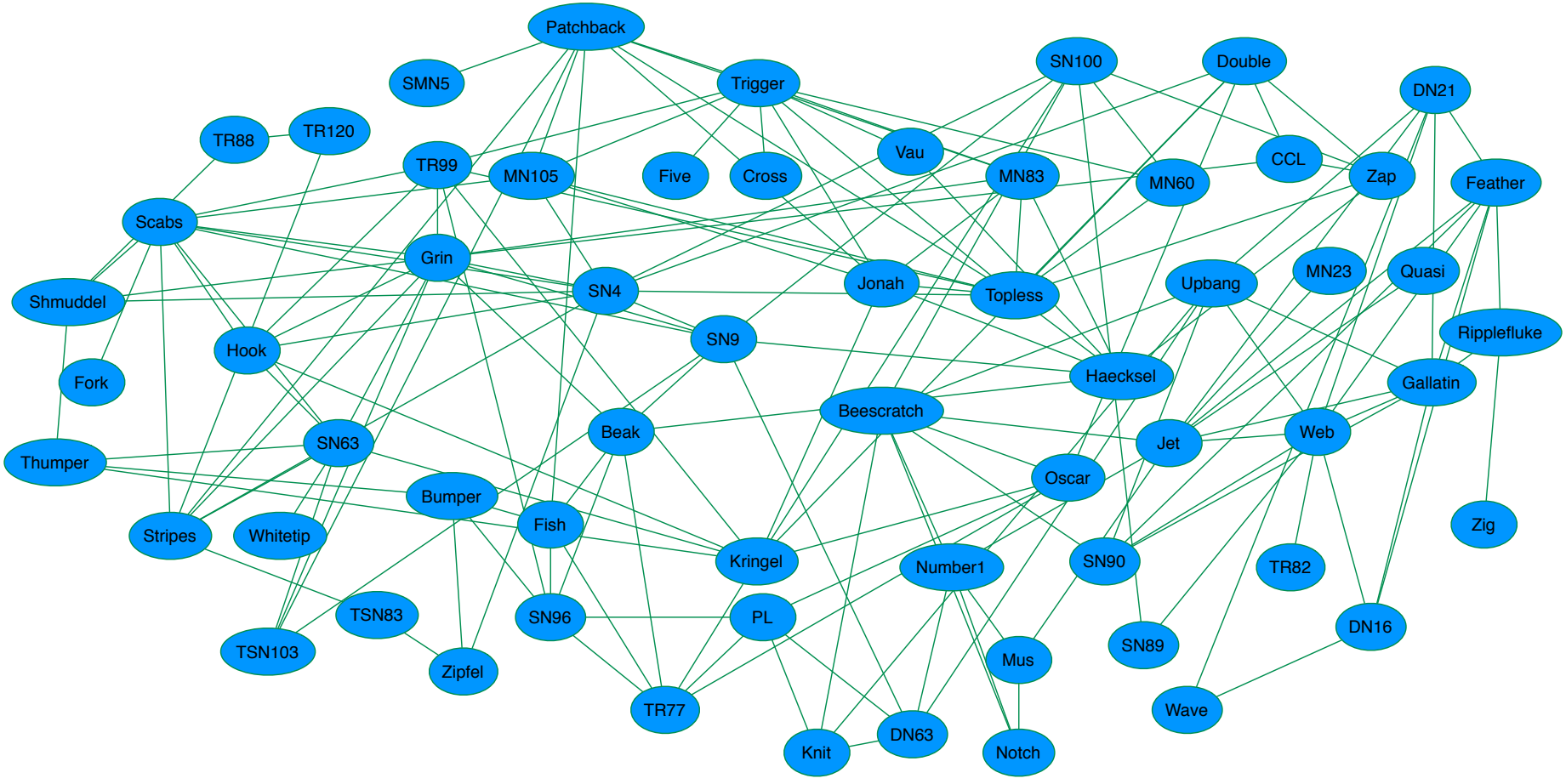


Is there a walk that traverses each bridge exactly once?

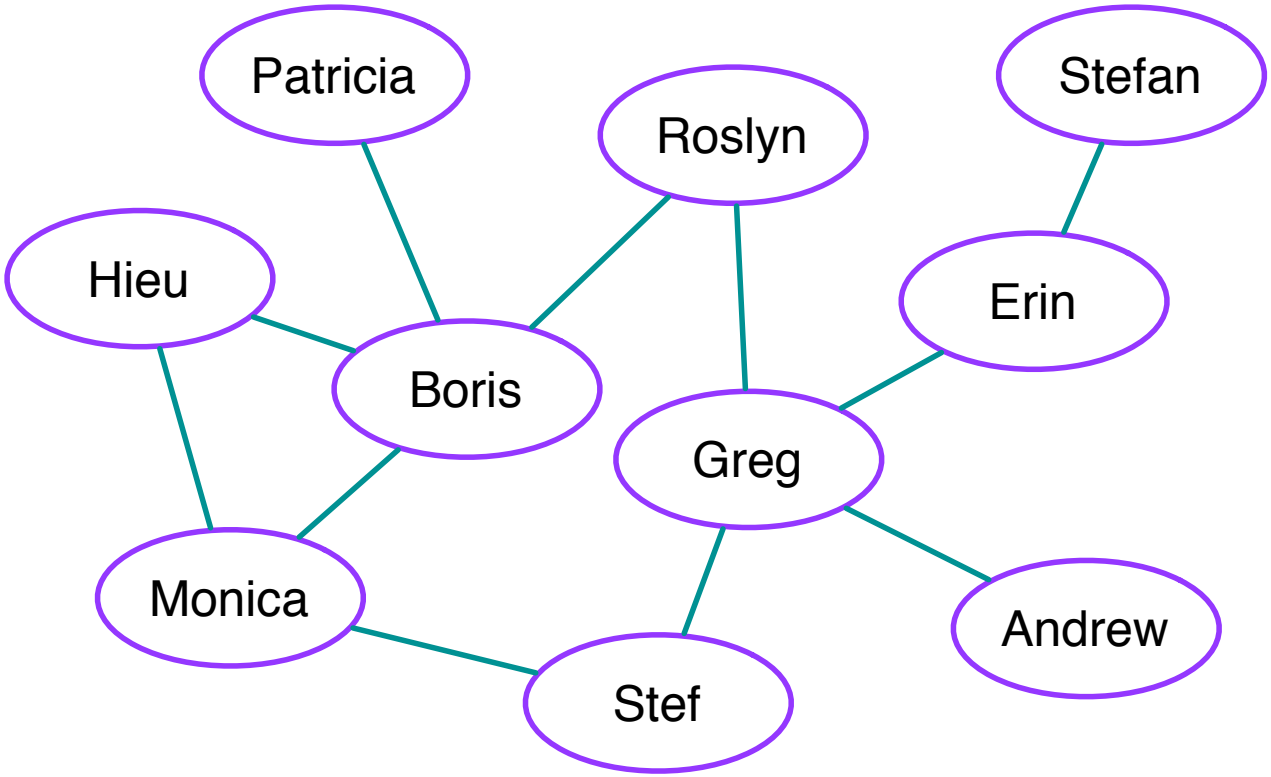
What is a graph?

- Vertices and edges.
- Nodes and links.
- People and relationships.

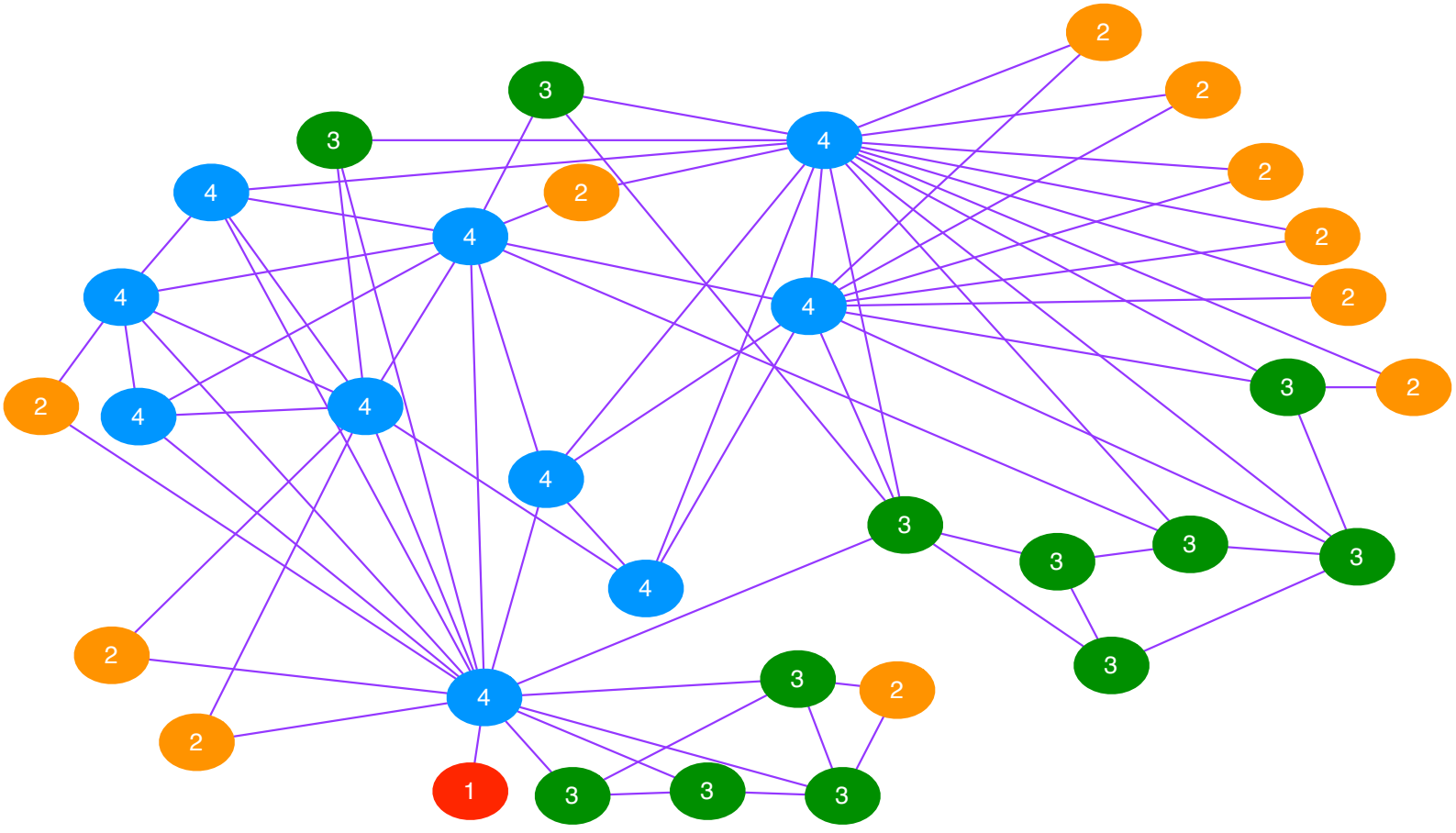
What is a graph? - Dolphin network



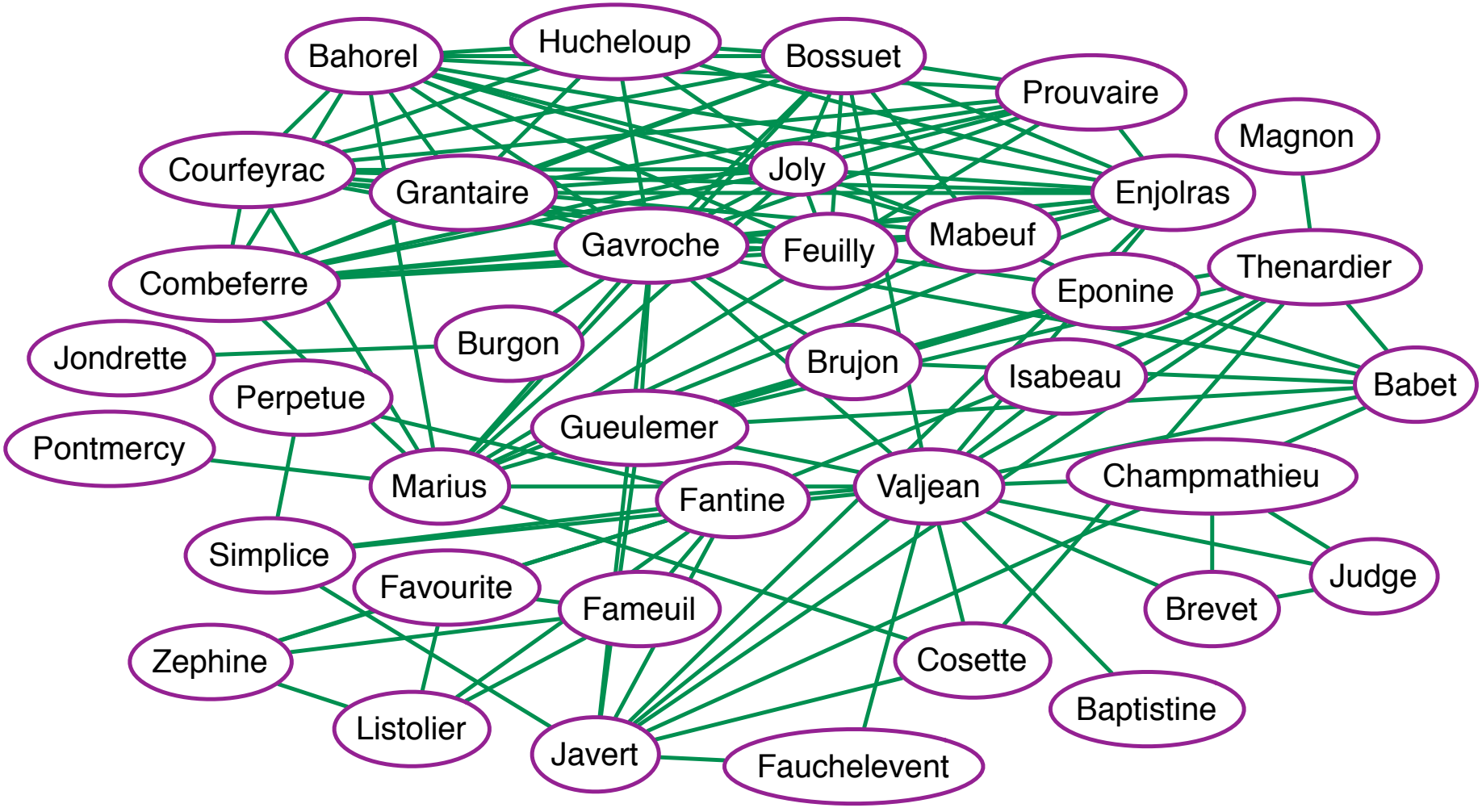
What is a graph? - Friends network



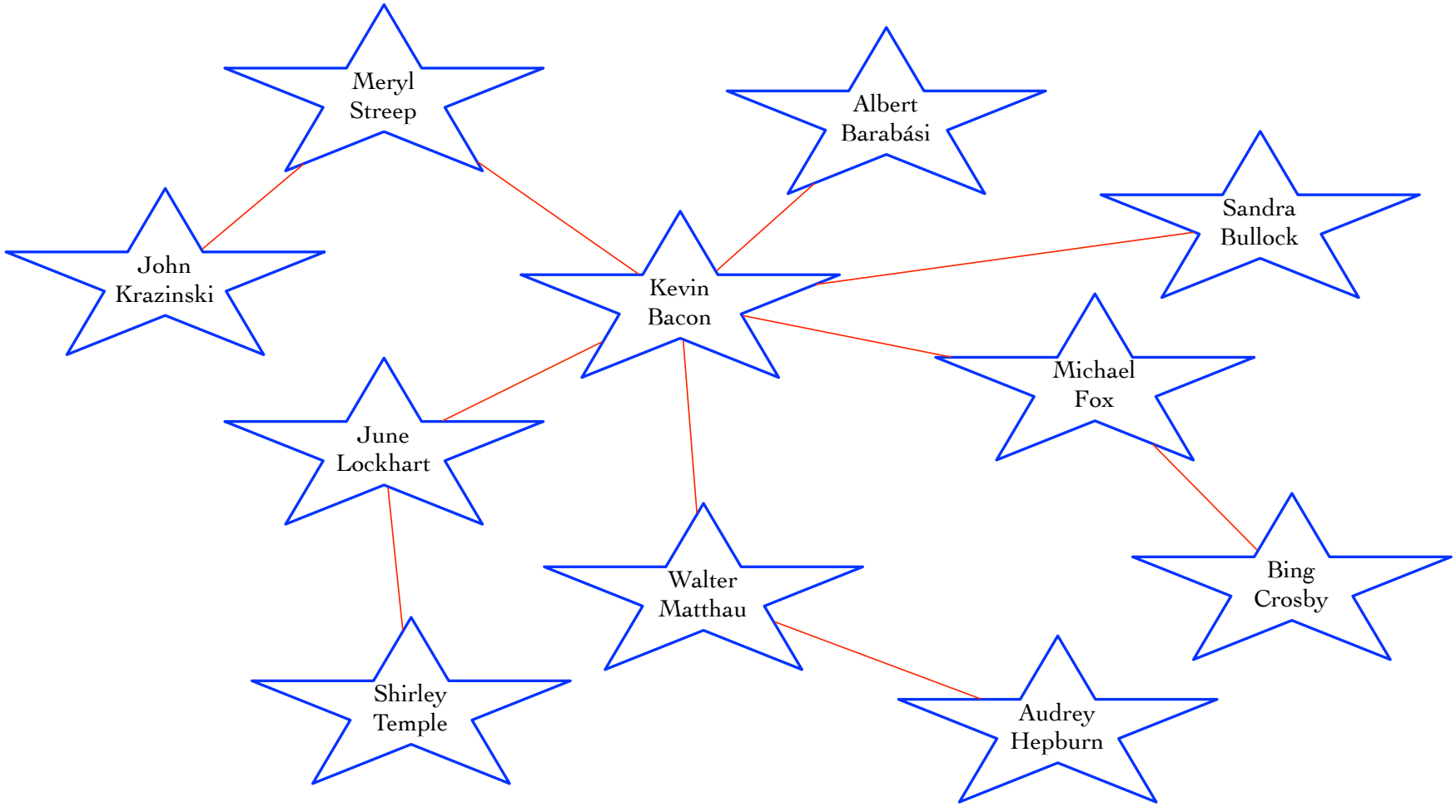
What is a graph? - Karate network



What is a graph? - Les Misérables network



What is a graph? - Actors network



Revisit the *Seven Bridges of Königsberg*

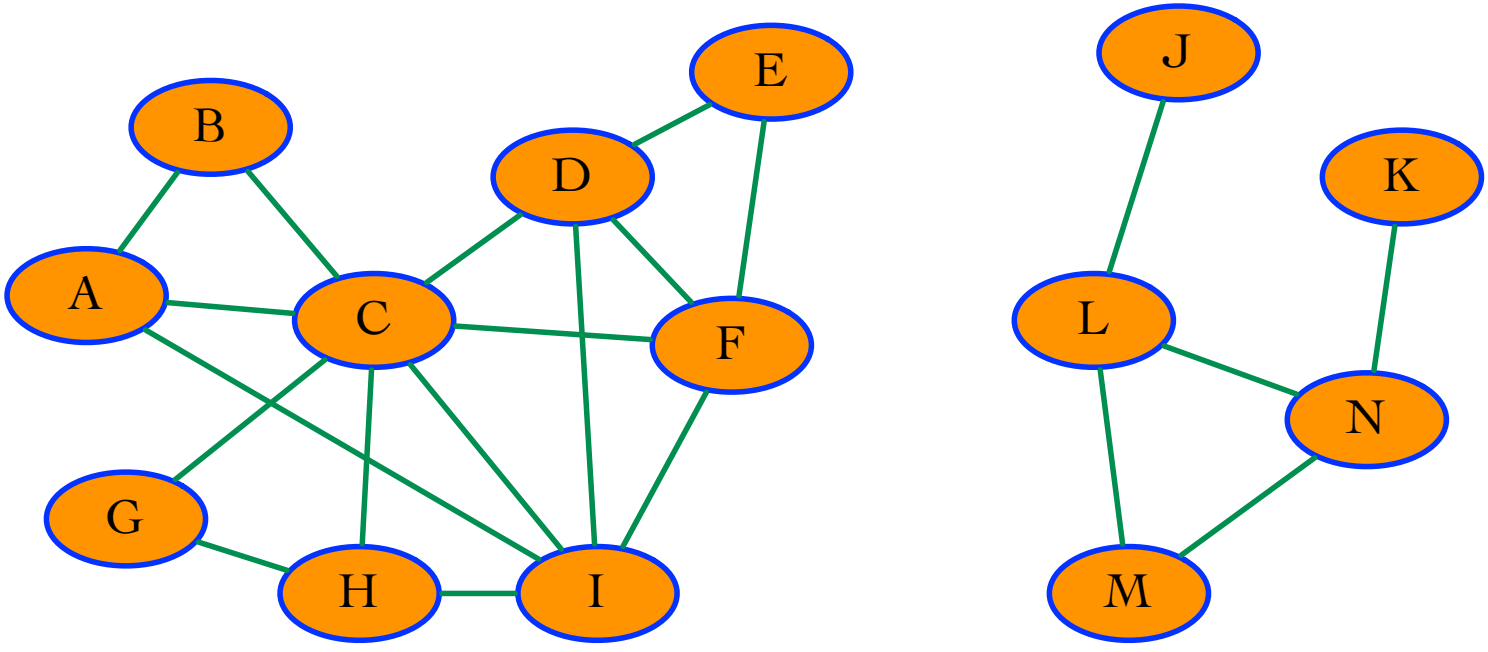
Theorem

There is a walk through a graph that traverses each edge exactly once if and only if the graph is connected and there are exactly two or zero vertices of odd degree.

Types of networks

- Collaboration networks
- Who-talks-to-whom graphs
- Information linkage graphs
- Technological networks
- Biological networks

Basic ways to describe a graph

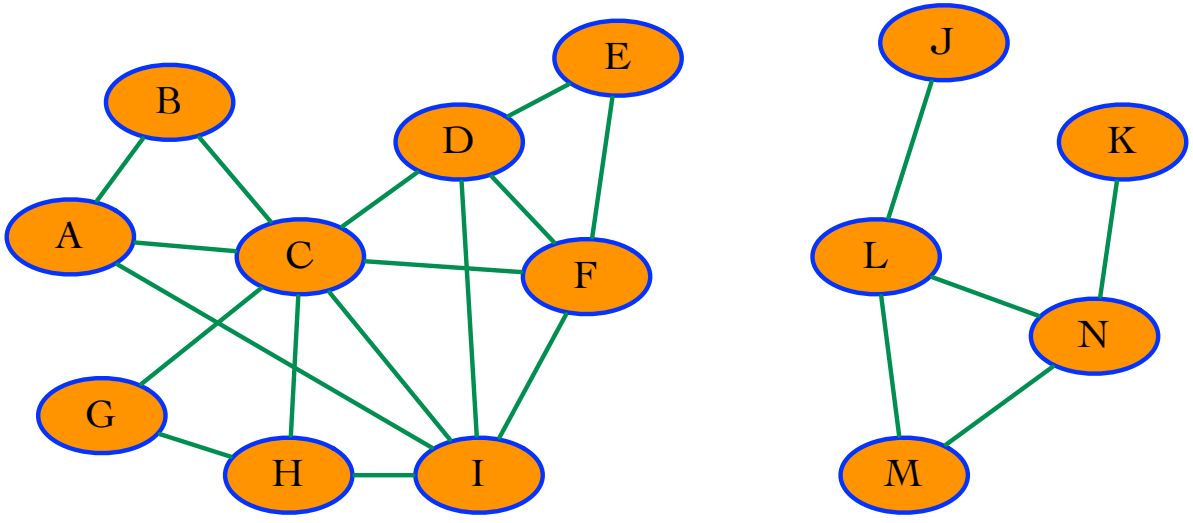


Definition

A vertex A and a vertex B are *neighbors* if there is an edge, AB, between A and B.

D is neighbors with E, F, and C, but not B.

Basic ways to describe a graph

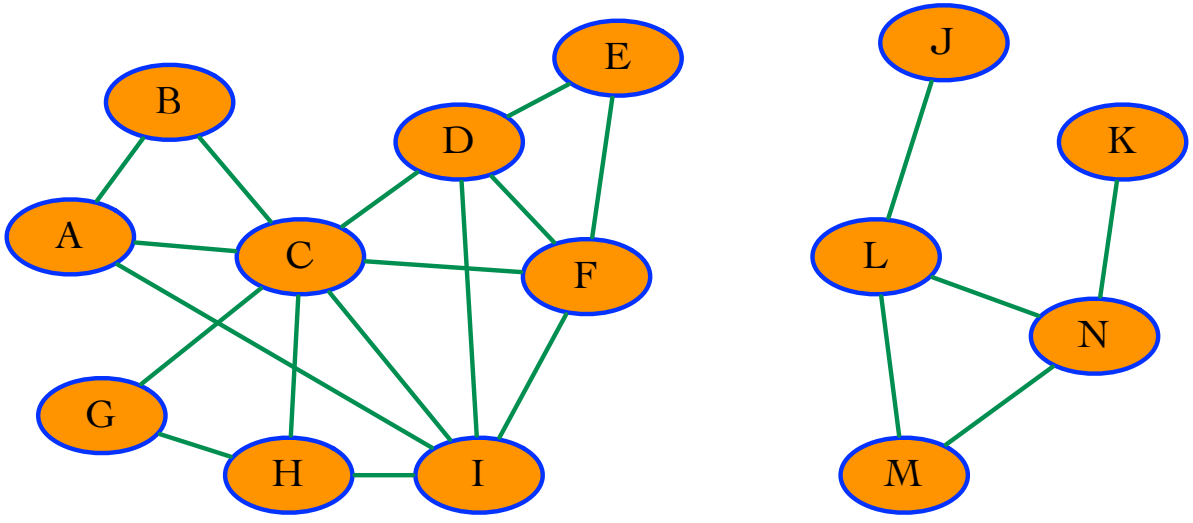


Definition

The *degree* of a vertex is the number of edges adjacent to it (or the number of neighbors).

C has degree 7. J has degree 1.

Basic ways to describe a graph

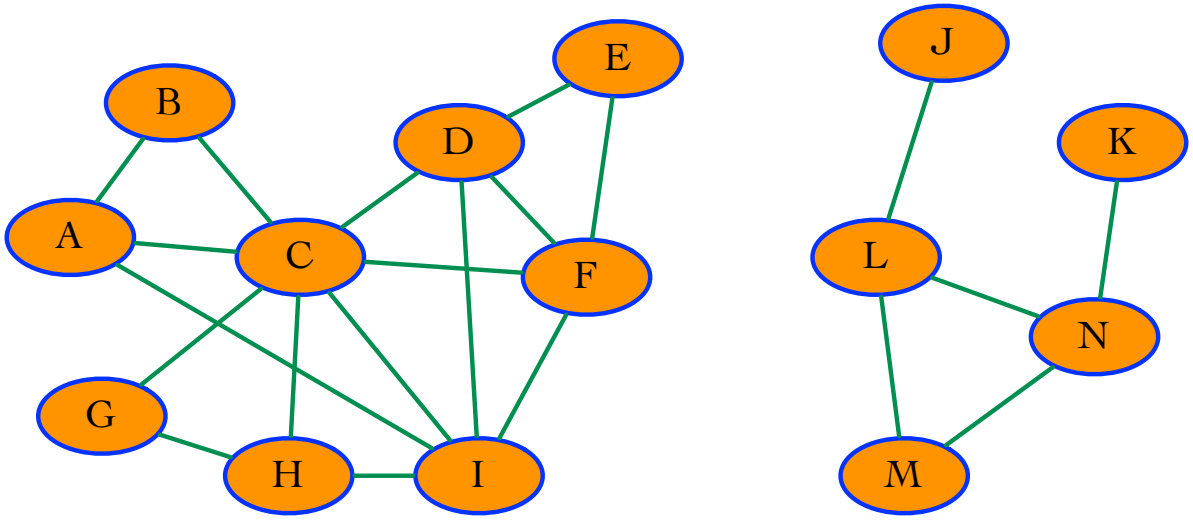


Definition

The *degree distribution* of a graph is the number of vertices of each degree.

$\{0, 2, 4, 4, 2, 1, 0, 1\}$ or $\{0, 1/7, 2/7, 2/7, 1/7, 1/14, 0, 1/14\}$

Basic ways to describe a graph

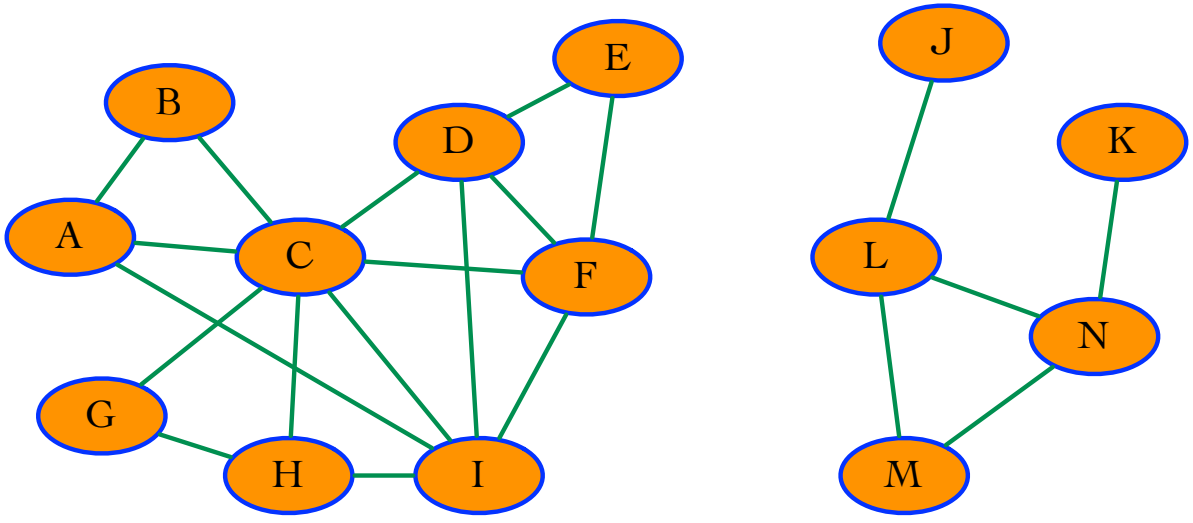


Definition

A *path* between two vertices is a sequence of vertices with the property that each consecutive pair in the sequence is connected by an edge.

There are many paths connecting A and E.
One of these is A, C, D, E, another is A, B, C, G, H, I, F, E

Basic ways to describe a graph

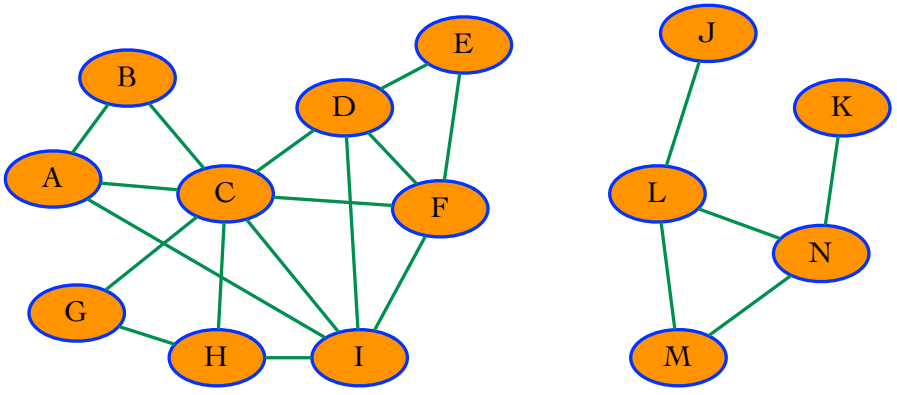


Definition

We say that a graph is *connected* if for each pair of vertices, there is a path between them.

The above graph is not connected.

Basic ways to describe a graph

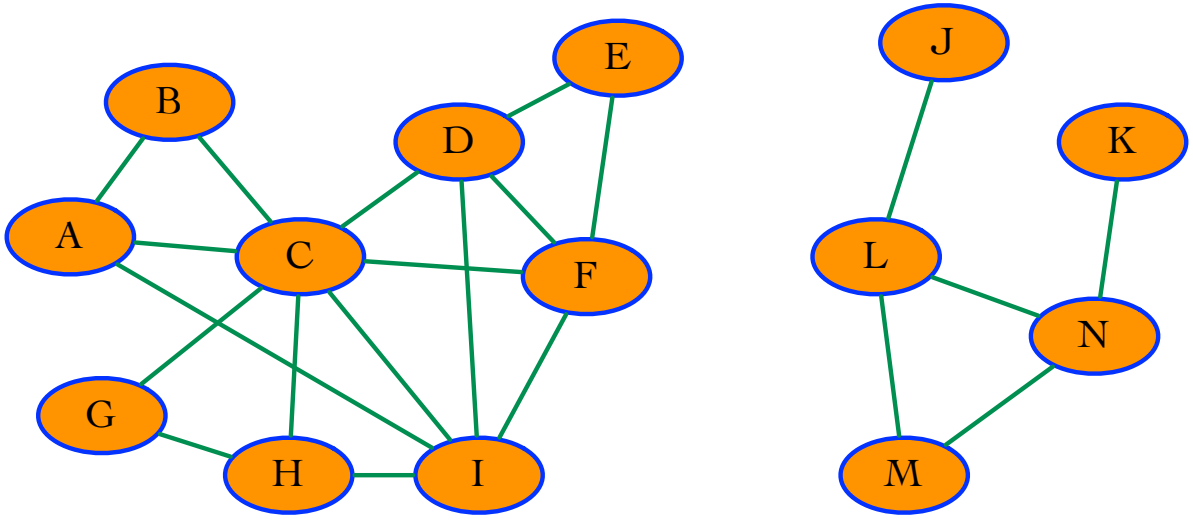


Definition

A *connected component* (or just component) of a graph is a subset of vertices such that every vertex in the subset has a path to every other vertex in the subset and the subset is not a part of some larger subset with the property that there is a path between every pair of vertices.

There are two components in the graph $A, B, C, D, E, F, G, H, I$ and J, K, L, M, N. Note that L, M, N is not a component.

Basic ways to describe a graph

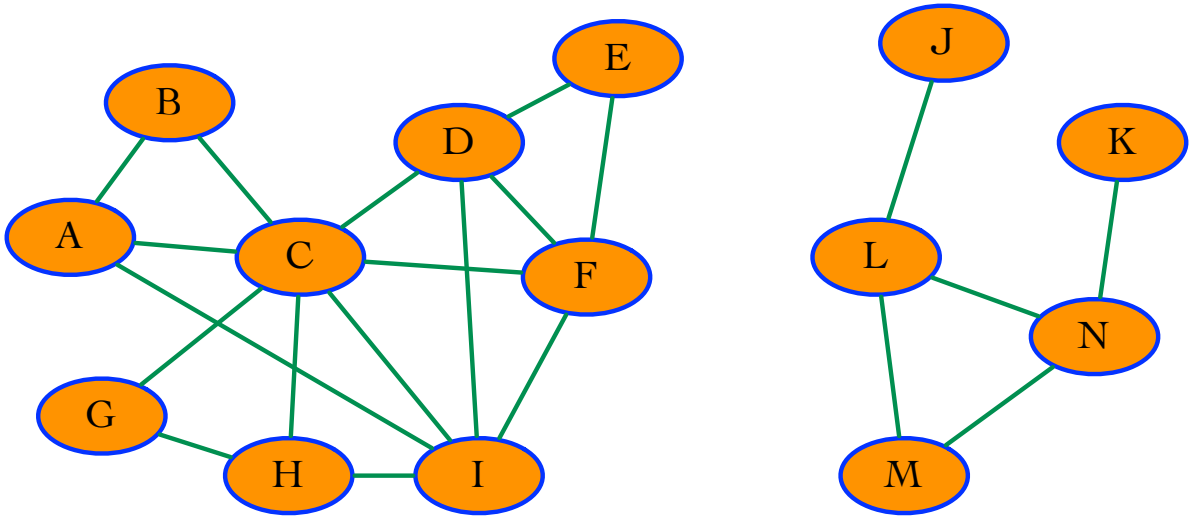


Definition

A *cycle* is a path with at least 3 edges in which the first and last vertices are the same, but otherwise all vertices are distinct.

L, M, N is a cycle, so is A, C, I, H, and many more...

Basic ways to describe a graph



Definition

The *distance* between two vertices is the length of the shortest path connecting them.

The distance between A and F is 2.
By convention, the distance between H and K is ∞.

Algorithms

- Wikipedia: "A step-by-step procedure for calculations"
- My undergrad advisor, Dr. Basor: "A recipe"
- al-Khwārizmī (780-850) was a Persian mathematician, astronomer, geographer and a scholar in the House of Wisdom in Baghdad. He was the most widely read mathematician in Europe in the late Middle Ages.

A statue of Al-Khwarizmi in Uzbekistan.

Algorithm for Calculating Distance

Breadth-First Search

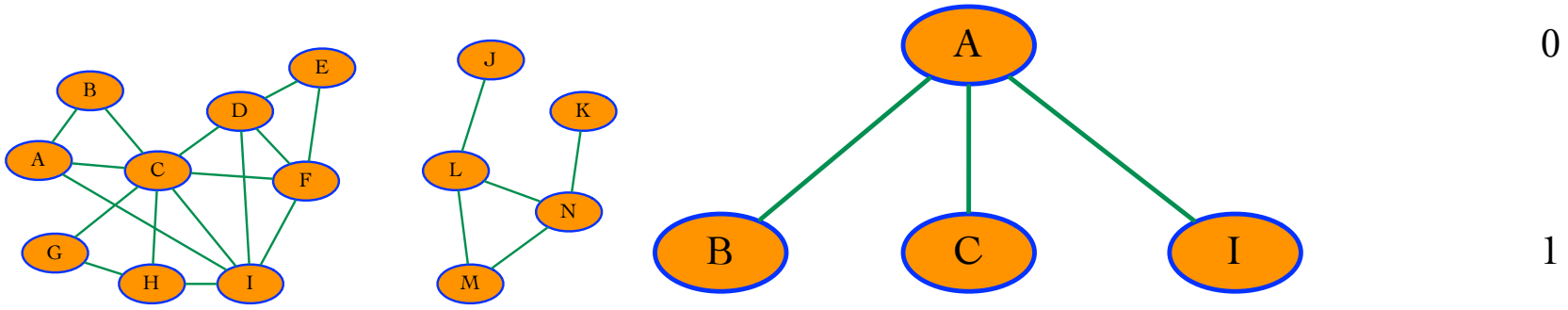
- Calculates the distance between a vertex and all of the other vertices in the graph.
- Runs quickly. $(O(|V| + |E|))$

Breadth-First Search

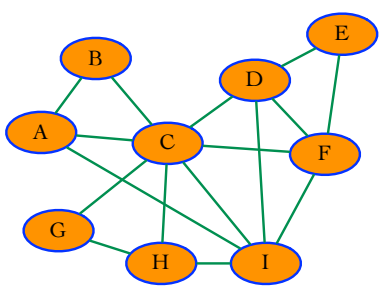
Algorithm

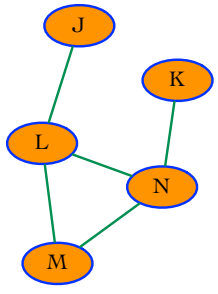
Input: a graph G and a vertex A
All neighbors of A are at distance 1 from A.
All neighbors of vertices at distance 1 from A (that are not themselves at distance 1) are at distance 2 from A.
All neighbors of vertices at distance i from A (that are not at distance $\leq i$) are at distance $i + 1$ from A.

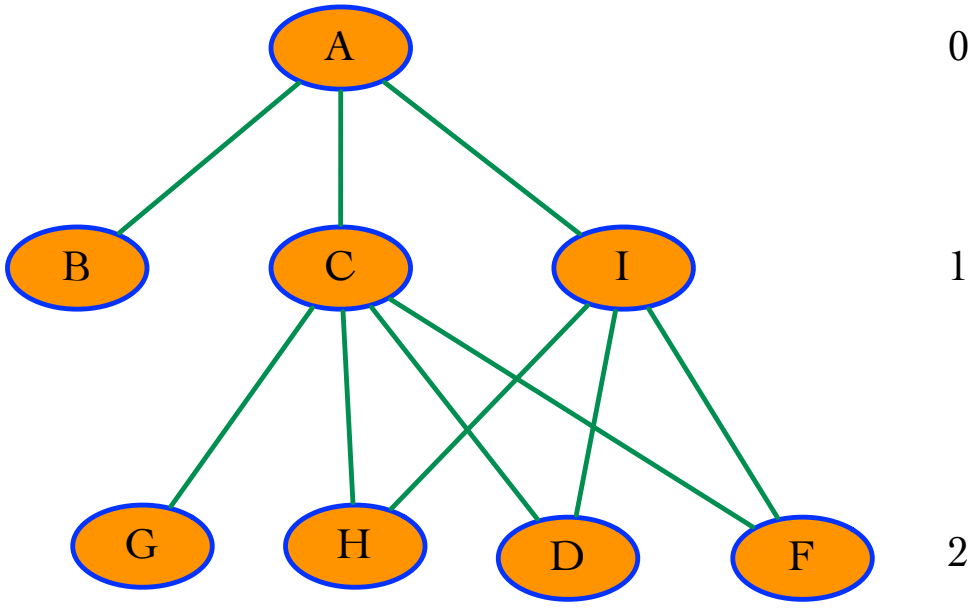
Breadth-First Search



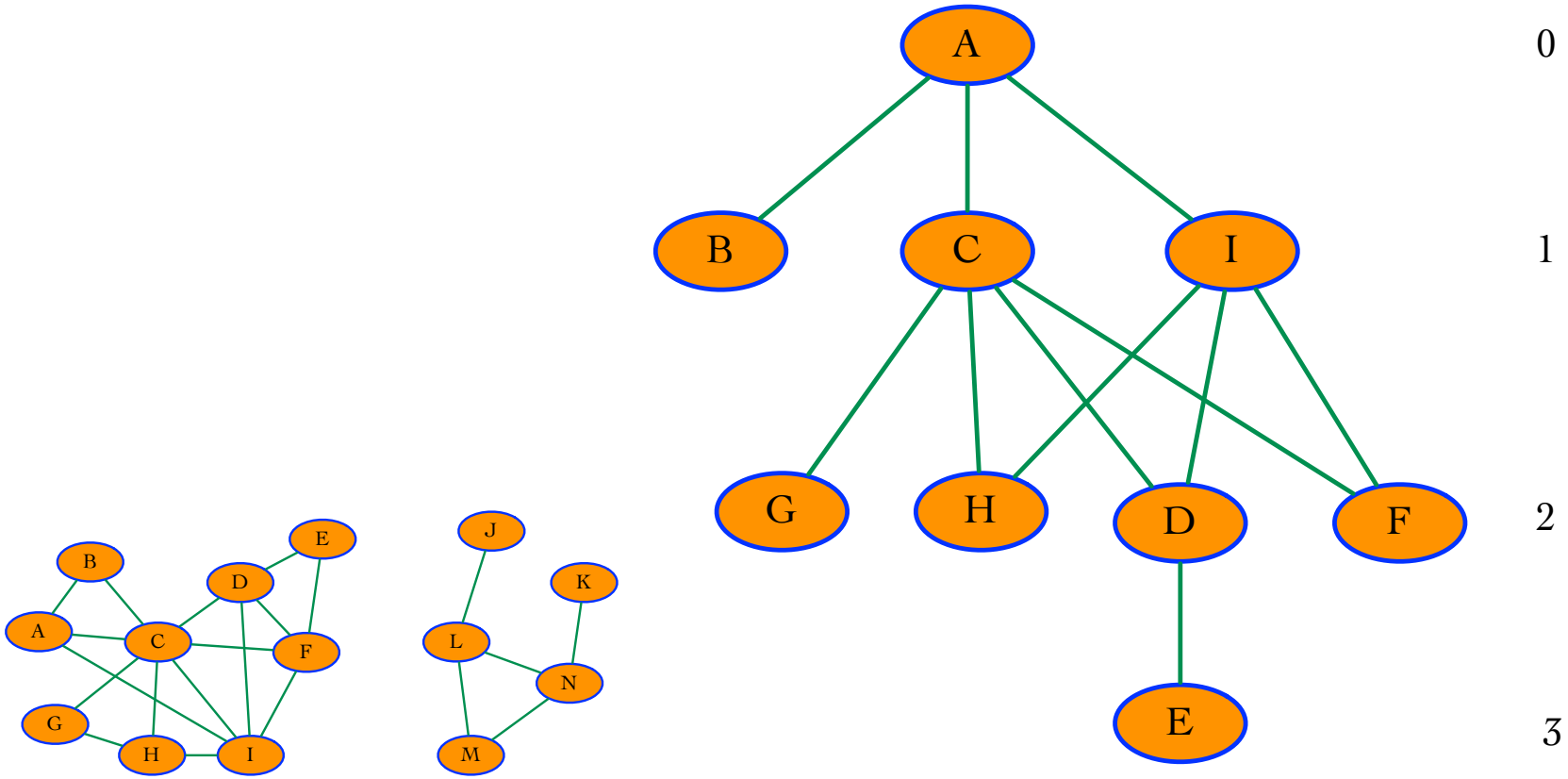
Breadth-First Search







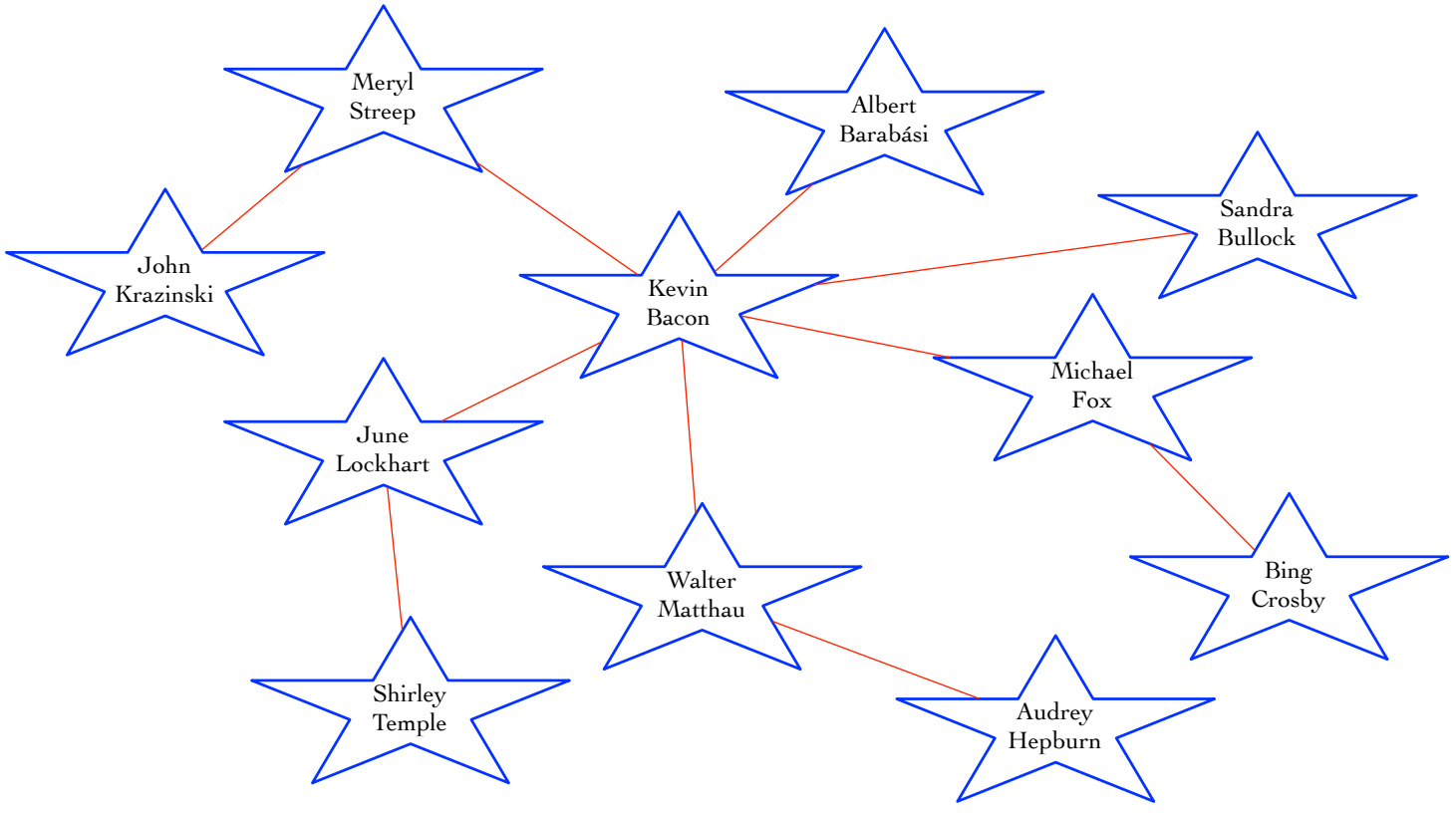
Breadth-First Search



Small worlds phenomenon

- Milgram experiment (1960's)
- 296 starters, 1 target (a stock broker in Boston)
- Median path length of 6.

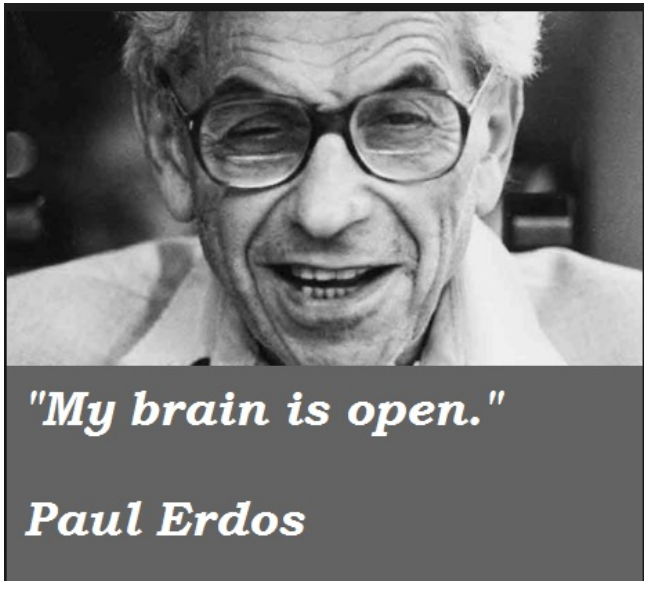
Hollywood small world



Average Bacon number is 2.9!

Erdős number

- Paul Erdős was a Hungarian mathematician (1913-1996).



- Most prolific mathematician of the 20th century.
- So prolific in fact that his friends designed the Erdős number.

Erdős number