Strong and Weak Ties

CMPS 17

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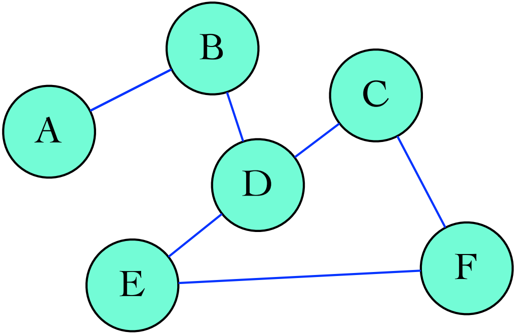
Strong and weak ties

- So far we have considered networks as static, when in fact most social networks are dynamic.
- As networks evolve over time:
  - Which vertices and edges enter the graph?
  - Which vertices and edges vanish?

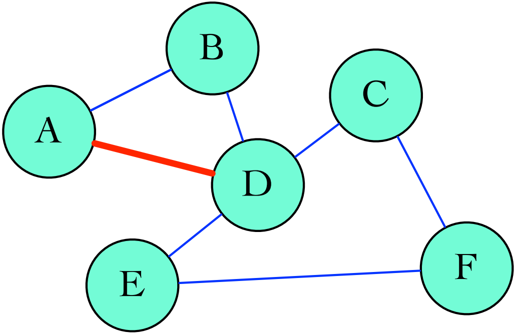
Triadic closure



Definition

Triadic closure is the principle that if A is connected to B and D is connected to B it is likely that A and D will also be connected.

Triadic closure



Definition

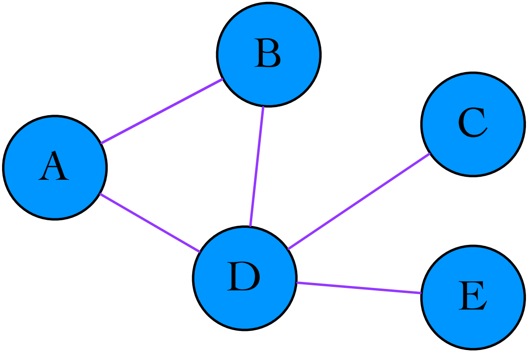
Triadic closure is the principle that if A is connected to B and D is connected to B it is likely that A and D will also be connected.

The clustering coefficient

Definition

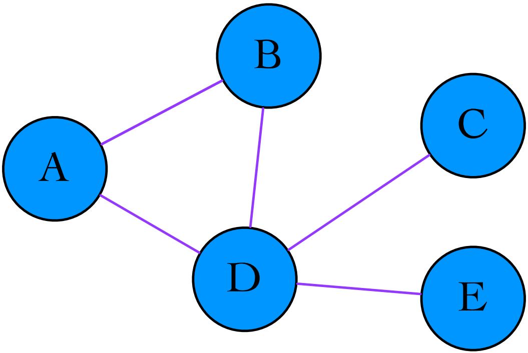
The *clustering coefficient* of a vertex A is the probability that two randomly selected neighbors of A are also neighbors.

The clustering coefficient



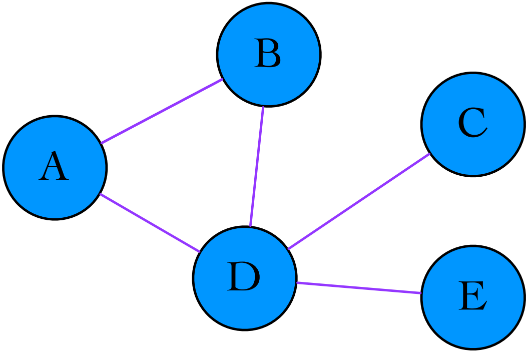
- D's clustering coefficient is $1/6$ because 1 of the six possible pairs of D's neighbors are connected.

The clustering coefficient



- The clustering coefficient of a vertex is between 0 (none of the vertex's neighbors are connected) and 1 (all of the vertex's neighbors are connected).

The clustering coefficient



- The clustering coefficient of a graph is the average over all vertices of their clustering coefficients. Here the clustering coefficient is approximately .83.

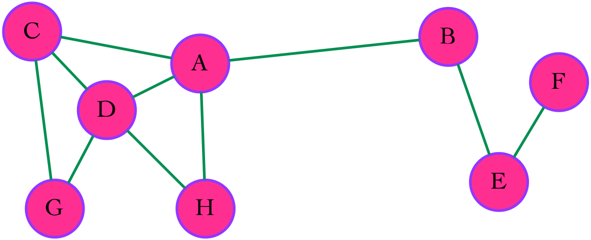
Explanations for triadic closure in friendship networks

- Opportunity to meet
- More trust if two people have a shared friend
- One is incentivized to bring two of one's friends together (raises one's clustering coefficient).
  - Teenage girls with a lower clustering coefficient in their social network of friends are significantly more likely to contemplate suicide than those with higher clustering coefficient.

Bridges

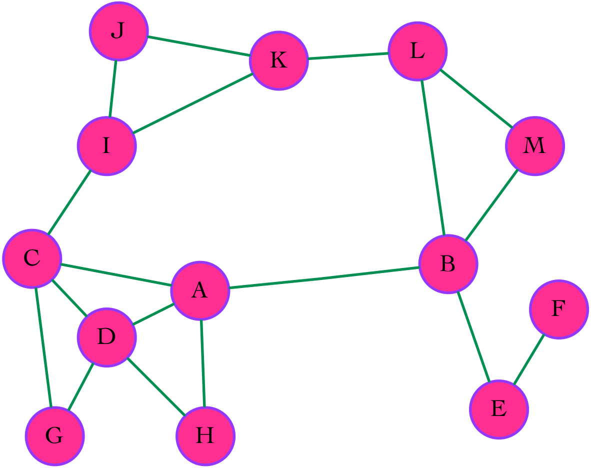
Definition

An edge that joins two vertices A and B in a graph is a *bridge* if removing the edge causes A and B to lie in different components.



Local bridges

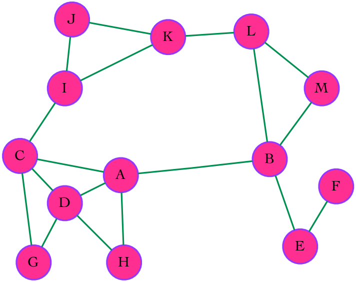
It seems rather unlikely that many bridges exist given how connected most real networks are.



Local bridges

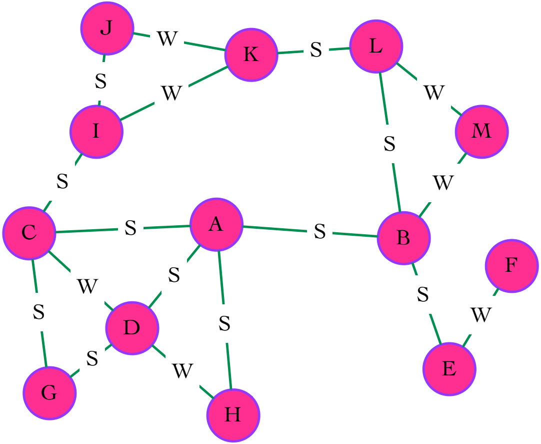
Definition

An edge joining A and B is a *local bridge* if A and B have no neighbors in common. The *span* of a local bridge is the distance between its endpoints if the edge was deleted.



Here edge AB is a local edge and the span of AB is 5.

Strong and weak ties

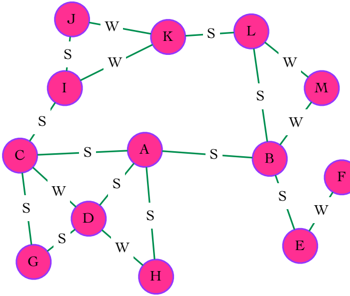


The definition of strong and weak will depend on the network.

Strong Triadic Closure property

Definition

We say that a vertex A violates the *Strong Triadic Closure* property if it has strong ties to two other vertices B and C, but there is no edge at all between B and C. Otherwise A satisfies the Strong Triadic Closure property.



Does G satisfy the Strong Triadic Closure property? B?

Connection between strong and weak ties and bridges

Theorem

If a vertex A satisfies the Strong Triadic Closure property and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie.

We can (will) prove this!

Connection between strong and weak ties and bridges

Theorem

If a vertex A satisfies the Strong Triadic Closure property and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie.

Proof.

We will proceed by contradiction:
Suppose that A satisfies the S.T.C. property. Further suppose A is involved in a local bridge with a strong tie B.
A must have a strong tie to some other vertex C.
Is there an edge from B to C?
Yes, since A satisfies the S.T.C. property.
No, since AB is a local bridge (must have no neighbors in common.) $\Rightarrow\Leftarrow$ Contradiction. □

Tie strength and bridges in real large networks

In order to analyze real networks with respect to ties and bridges, we will soften the definitions.

Definition

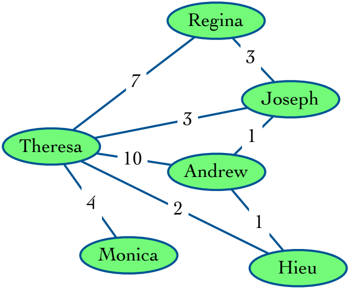
We define the *tie strength* of an edge to be a numeric value rating how strong the tie is.

Generalization of tie strength

Definition

We define the *tie strength* of an edge to be a numeric value rating how strong the tie is.

For example, in a phone network, if I call my friend 10 times in a one month period, the tie strength between us will be 10.



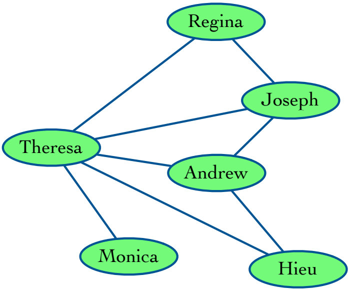
Generalization of local bridges

Definition

The *neighborhood overlap* of an edge AB is the ratio:

$$\frac{\text{number of vertices who are neighbors of both } A \text{ and } B}{\text{number of vertices who are neighbors of at least one of } A \text{ or } B}$$

In the denominator we don't count A or B.



For example, the neighborhood overlap for the edge connection Andrew and Theresa is $2/4 = 1/2$

Generalization of local bridges

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Theorem

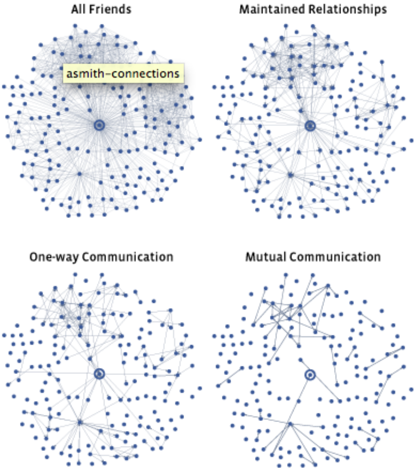
An edge is a local bridge if and only if it has neighborhood overlap 0.

Tie strength on Facebook

Can someone really have hundreds of friends? Studied by Cameron Marlow at Facebook. Three categories of "friends" -corresponding to tie strength.

- *Reciprocal (mutual) communication*: user both sent and received messages from friend.
- *One-way communication*: user sent messages to friend.
- *Maintained relationship*: user followed information about friend.

Tie strength on Facebook



Describing vertices

Definition

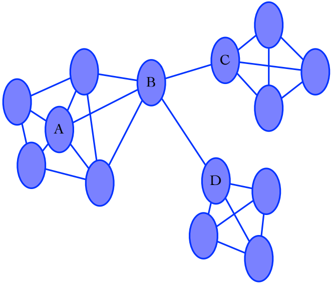
The *embeddedness* of an edge AB is the number of neighbors common to both A and B.

- This is the numerator in the definition for neighborhood overlap.
- If an edge has embeddedness 0, then it is a local bridge.

Embeddedness

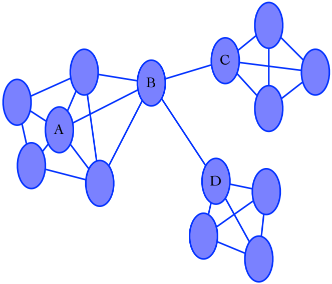
Definition

The *embeddedness* of an edge AB is the number of neighbors common to both A and B.



The embeddedness of AB is 2 because A and B have two neighbors in common. In fact all of the edges that A is adjacent to have significant embeddedness. Easier for trust to take place between the vertices of an embedded edge.

Embeddedness



The endpoints of the embedded edge are less likely to cheat one another because of the social consequence.

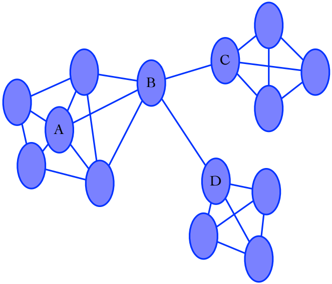
Not so for edges with 0 embeddedness.

It is riskier for B to deal with C and D than A.

Describing vertices

Definition

A *structural hole* is an "empty space" in the network between two sets of vertices that don't otherwise interact closely.



Here B spans a structural hole. This offers some distinct advantages.