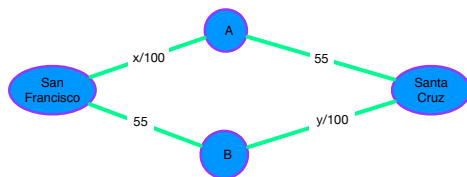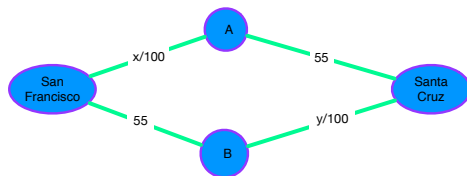# Traffic

Traffic

Traffic

Suppose that there were two possibilities for getting from San Fransisco to Santa Cruz:

There are 5,000 people that need to get from San Francisco to Santa Cruz each morning.
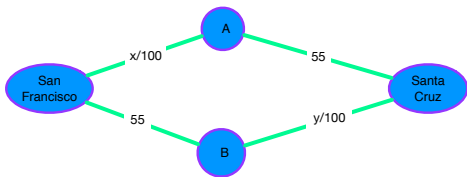
How should they travel?

Traffic

We can think about this network as a game. There are 5,000 players and each have 2 strategies: *A* or *B*.

If 4,000 drivers take route *A* those drivers will have a travel time of 95 minutes. The remaining 1,000 drivers will have a travel time of 65 minutes.

If 2,500 drivers take route *A* and 2,500 drivers take route *B*, the travel time for each driver will be 80 minutes.
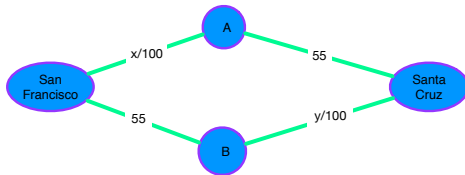
Defining the Payoff Matrix

- ☐ Now suppose that there are just two players (drivers).
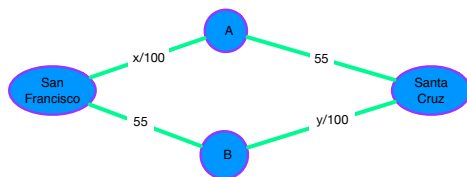- ☐ The following payoff matrix defines the game:

		Driver 2	
		Through A	Through B
Driver 1	Through A	$-55.02, -55.02$	$-55.01, -55.01$
	Through B	$-55.01, -55.01$	$-55.02, -55.02$

Traffic - Nash Equilibrium

- ☐ Is there a Nash equilibrium for this game?
- ☐ Any set of strategies for all 5,000 drivers that split the routes in half is a Nash equilibrium. No driver can do better by choosing a different route.
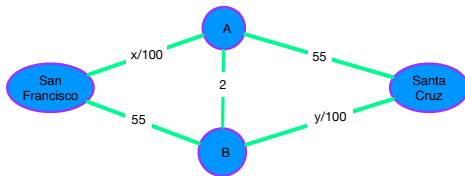
Traffic - Nash Equilibrium

□ How can we make this formal?

   □ Suppose x people take the route through A and y people take the route through B.

   □ The travel time for a person on the route through A is $x/100 + 55$.

   □ The travel time for a person on the route through B is $y/100 + 55$.

   □ Set these two values equal and solve for x and y. This gives the Nash equilibrium.

Add a road?

Suppose that we added a road to try to help ease congestion:

- □ Now there are four routes.
- □ Is there a Nash equilibrium here?
  - □ Yes! Everyone must go to A, along the new road to B. Now everyone's travel time is worse! 102 minutes.
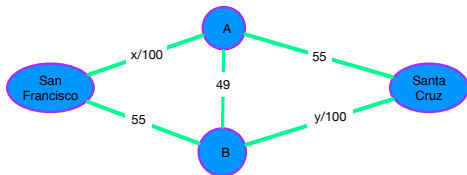- □ This is known as Braess's paradox

Braess's paradox

- Named after mathematician Dietrich Braess.

- Adding more paths in networks does not necessarily reduce congestion.

- Shown time and again in the real world. The most vivid example is when a 6 lane highway was shut down in Korea to build a park and commute times decreased.

What is the Nash equilibria with 1,000 drivers?