

Binary Tree Traversal

Traversal

Definition

Tree traversal is a process in which each vertex in a tree is visited.

We will focus our attention on four types of traversal:

- Pre-order
- Post-order
- In-order
- Level-order

The first three traversals are considered *depth-first* traversals, while the last is considered a *breadth-first* traversal.

Pre-order Traversal

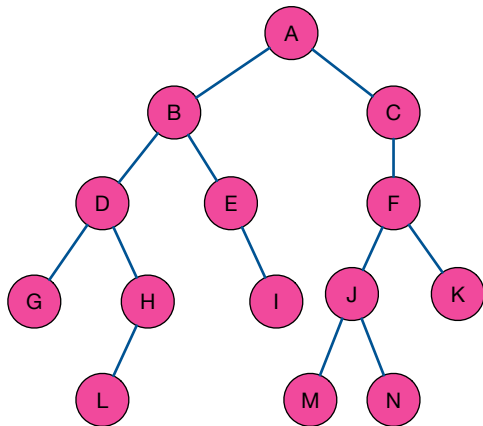
Pre-order(T, v)

Input: A tree, T , and a vertex v (initially the root).

Output: A list of the vertices in T (in pre-order).

- 1: **if** $v = \text{null}$ **then**
 - 2: Return
 - 3: Visit v
 - 4: Pre-order($T, v.\text{left}$)
 - 5: Pre-order($T, v.\text{right}$)
-

Pre-order Traversal



Post-order Traversal

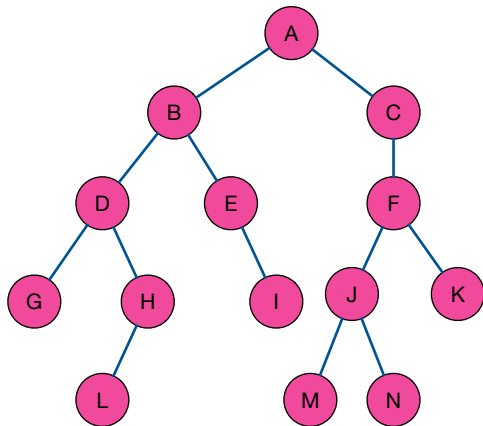
Post-order(T, v)

Input: A tree, T , and a vertex v (initially the root).

Output: A list of the vertices in T (in post-order).

- 1: **if** $v = \text{null}$ **then**
 - 2: Return
 - 3: Post-order($T, v.\text{left}$)
 - 4: Post-order($T, v.\text{right}$)
 - 5: Visit v
-

Post-order Traversal



In-order Traversal

In-order(T, v)

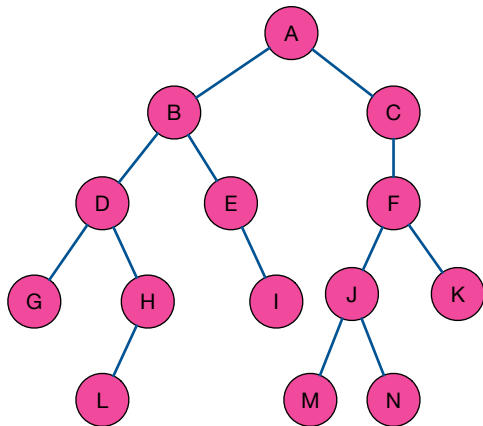
Input: A tree, T , and a vertex v (initially the root).

Output: A list of the vertices in T (in in-order).

- 1: **if** $v = \text{null}$ **then**
 - 2: Return
 - 3: In-order($T, v.\text{left}$)
 - 4: Visit v
 - 5: In-order($T, v.\text{right}$)
-

For a binary search tree, in-order traversal gives vertices in non-decreasing order.

In-order Traversal



Level-order Traversal

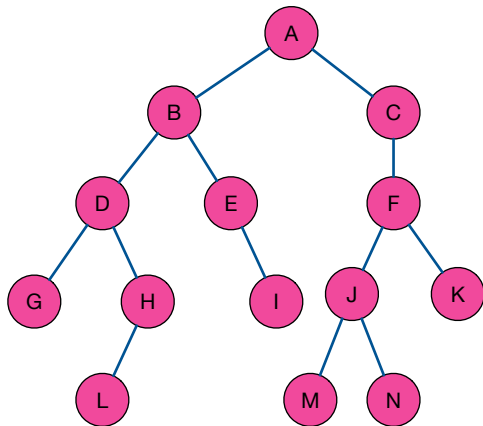
Level-order(T, v)

Input: A tree, T , and a vertex v (initially the root).

Output: A list of the vertices in T (in level-order).

- 1: Let Q be an empty queue.
 - 2: $Q.enqueue(v)$
 - 3: **while** $Q \neq \emptyset$ **do**
 - 4: $x = Q.dequeue$
 - 5: Visit x
 - 6: **if** $x.left \neq null$ **then**
 - 7: $Q.enqueue(x.left)$
 - 8: **if** $x.right \neq null$ **then**
 - 9: $Q.enqueue(x.right)$
-

Level-order Traversal



Time complexity

What is the time complexity for the first three traversal methods?

- Each function is called once for each vertex.
- $O(n)$

What is the time complexity for level-order traversal?

- Each vertex is pushed onto the queue once and popped off the queue once.
- $O(n)$

Reconstruction

Suppose you had a graph with 14 vertices, A, B, \dots, N . You wrote down the pre and post orderings for the vertices:

Preorder: $A, B, D, E, H, L, M, I, F, C, G, J, K, N$

Postorder: $D, L, M, H, I, E, F, B, J, N, K, G, C, A$

Then you lost your graph. Can you reconstruct the graph?